How can we learn causal structures?

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Most people think that science explains why things happen. This generalized perception, common even among people with some scientific training, is psychologically justified by the fact that we are surrounded by causal systems. Almost all artefacts that we use in our daily life are causal systems that respond to commands. Cars, for instance, have a starting key, brakes, accelerator, steering wheel plus a number of buttons and knobs to select functions that range from controlling the temperature to plan the route for reaching a destination. Even home appliances exhibit a number of functions that can be selected.

Since antiquity philosophers have debated the meaning of causation and different definitions of causation have been proposed. However, in the modern highly technological world, it is reasonable to identify causation with manipulability. We say that a variable X has a causal effect on another variable Y if after a change of X we find a change of Y but not vice versa. A causal system exhibits input variables which control but are not controlled by output variables. A barometer offers an example of causation often cited in the literature. If the atmospheric pressure changes, then the needle of the barometer moves but if we move artificially the needle of the barometer the atmospheric pressure does not change at all. The variable atmospheric pressure has a causal effect on the position of the needle but the position of the needle has no causal effect on atmospheric pressure.

We are surrounded by systems, either human artefacts or natural systems, were input variables control the value of output variables. Therefore, it is easy to think that science gives causal explanations of physical events. Probably we can even say that the success of science and technology has partially changed the attitude of people relative to human affairs. Now we tend to have an engineering view of everything, we tend to think that there is an engineering solution to most problems.

But reality is not so simple. Science is not causal, it is observational. This means that basic science is made by laws of nature that describe observed reality. But we need to be more precise about the concept of science. In 1948, Warren Weaver published the article Science and Complexity[1] In this article Weaver anticipated that there are three levels of science:

1. The science of simple trajectories, that he identified with classical physics
2. The science of disorganized complexity, that he identified with statistical mechanics
3. The future science of complexity.

The current view is that science is hierarchical in the sense that there are different levels of science that cannot be reduced to a basic universal science. Each level needs specific laws and principles. In his 1972 article More is different,[2] Philip Anderson, recipient of the 1977 Nobel Prize in Physics, attacked the notion of reductionism and claimed that there is a hierarchy of

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[1] Warren Weaver, American Scientist, Vol. 36, No. 4 October 1948, pp. 536-544.
[2] P. W. Anderson, More Is Different, Science, New Series, Vol. 177, No. 4047. Aug. 4, 1972, pp. 393)

physical theories. Modern science deals with basic laws of nature, but it also studies complex systems such as biological systems.

Basic laws of physics, typically formulated with differential equations, are not causal but descriptive. At the level of basic laws, physics includes a descriptive framework of variables and laws that describe the evolution of variables. The behavior of physical systems is obtained with logical deductions from basic laws. For example, the theory of electromagnetism is formulated by a set of differential equations, the Maxwell equations. Within the limits of classical physics, the Maxwell equations describe the time and space evolution of electromagnetic fields. If we want to know how electromagnetic fields behave in specific systems, for example in a microwave oven, we solve the equations with appropriate initial and boundary conditions.

But physics also studies systems, including complex systems. Systems might be causal because of their internal structures. Causal systems might be human artefacts, designed to be causal. For example, the braking subsystem of a car is a causal system. But natural systems can also be causal. A tree is a causal system as its development depends on the input of water and nutrients into the tree. All these physical causal systems respect the non-causal physical laws. However, their design is such that changing the input variable changes some output variable.

However, there are systems whose behavior is not well known. These systems include the economies, financial markets, social systems, firms. Our knowledge of economic, financial, and social systems is often obtained through a process of learning. However, we have only one realization of these systems that are by nature evolving. Statistical learning of evolving systems is in itself problematic.

Medicine is in between. Our knowledge of the functioning of the human body is only partial and the effects of drugs on the human body is not really known. In addition, in all these systems we cannot make experiments. We cannot restart the economy with different conditions, we cannot make experiments with firms, and we cannot experiment freely the effects of drugs.

In the last 35 years, from the beginning of the nineties, philosophers and computer scientists have tried to develop mathematical models of causation and algorithms to learn causal structures. In particular, we can cite the group at Carnegie Mellon University in Pittsburgh who developed TETRAD, a set of algorithms to represent and discover causal systems, and the American-Israelian Judea Pearl.

The critical issue that we want to address is the following: How can we learn the causal structure of a system of random variables of which only one realization is known? Consider, for example, an economy or a financial system. In most cases economic systems are described by a set of time series or by a set of continuous time stochastic processes.

In the Eighties, Cristopher Sims, proposed to use Vector Auto Regression (VAR) models. For his work Sims shared with Thomas Sargent the 2011 Nobel Memorial Prize in Economics. VAR models are observational, not causal. In fact, the general form of a VAR model is the following:

$$Y(T)=B0+B1Y(t-1)+\ldots+BpY(t-p)+U,$$

where the Y and U are time-dependent random vectors and an the Bs are coefficients. A VAR model is not causal. The relationships between the Y at different times are correlations. In fact a VAR model represents the behavior of observed time series. Clive Granger in his 1969 article

Investigating Causal Relations by Econometric Models and Cross-spectral Methods [3]proposed a causality test to determine if and the variables Y cause future values of the Y. For his work, Granger shared with Engle the 2003 Nobel Memorial Prize in Economics.

However, it was observed that the Granger test is a test of predictability not of causation. The point is that if the correlations between present and future values of Y are due to a confounder, that is an exogenous variable that causes both present and future values of Y. If there is a confounder, intervening by changing a variable at time t has no effect on variables at time t+1.

More in general, if we know the probability distribution or the matrix of correlations or covariances between a set of variables, how can we discover the causal structure of these variables, if any? Let's remark that correlations are observational quantities. That is, we compute correlations by observing the behaviour of the variables without any intervention. A positive correlation implies that the variables move together. If we compute the empirical correlation coefficient, we observe the movement of the variables as it happens in nature.

Causation is a different concept it implies that if we intervene changing the value of a variable the other variable will also change. But if we only know correlations, we do not have any solid argument to infer causation.

Therefore, how do we infer causation? Let's recall that we can represent causation through a graph. A graph is a set of nodes connected by edges. Edges might be directed, that is, might have a direction. Suppose nodes represent variables and directed edges represent causal relationships. Let's identify nodes with the corresponding variables. We stipulate that if nodes X and Y are connected by an edge directed from X to Y, then X causes Y. A graph is called a Directed Acyclic Graph (DAG), if there is no closed path of edges. A DAG therefore represent a causal system. Given a node X, the Parents of X, indicated as PA(X), are the set of all nodes with a directed connection to X. That is, PA(X) is the set of nodes that cause X.

Discovering causal structures is based on the following principle: Given a system described by variables X1,...,Xn, if the system has a causal structure than the joint probability distribution of the variable can be factorized as the product of conditionally independent variables:

$P(X1,\ldots,Xn) = \Pi P(Xi/PA(Xi))$ Π.

Each conditional probability $P(Xi/PA(Xi)$ represent a causal mechanism.

The discovery of causal structures is based on the property that if a causal structure represented by a graph *G* exists, then the joint probability distribution of variables can be factorized as $P(X1,\ldots,Xn) = \Pi P(Xi/PA(Xi))$. We have to note explicitly that the reverse does not hold in general. Discovery algorithms assume that the factorization of the joint probability distribution in a set of independent conditional distributions is a necessary and sufficient conditions for causality.

In summary, algorithms that discover causal structures work by testing properties such as factorization of the joint probability distribution into independent conditional probabilities that hold if there is a causal structure. If the algorithm can determine the set of independent conditional probabilities, then it is assumed that the algorithm has discovered a causal structure.

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[3] Granger, C. W. J. (1969). "Investigating Causal Relations by Econometric Models and Cross-spectral Methods". *Econometrica*. **37** (3): 424–438)